system records transit times and other information required for computing the elastic moduli.

5) Applications are primarily related to problems in determining basic knowledge about rock properties and the elastic behavior of rock systems under specified subsurface environmental conditions.

6) P-wave and S-wave velocities measured in samples of aluminum, Solenhofen limestone, and Boise sandstone are in good agreement with reliable published values.

Appendix

Angular Relationships at a Liquid–Solid Interface

When a P wave is incident in a liquid upon a plane liquid–solid interface, the relationships between the angle of incidence and the angles of refraction for P waves and S waves are given by Snell's Law:

$$\sin \alpha / \sin \epsilon = V_1 / V_p \quad (1)$$

$$\sin \alpha / \sin \beta = V_1 / V_s. \quad (2)$$

Amplitude Ratios

Amplitude ratios of elastic waves reflected and refracted at solid–liquid interfaces in the dual-mode ultrasonic apparatus were computed from equations published by Ergin [25]. Equation (3) gives the amplitude ratio c of the reflected P wave when a P wave is incident in a solid (aluminum) against a liquid (oil).

$$c = \frac{\cos \eta (m_1 \cos^2 2\zeta - 1/m_1 \sin 2\theta \sin 2\zeta) - m_1 n_1 r_1 \cos \theta}{\cos \eta (m_1 \cos^2 2\zeta + 1/m_1 \sin 2\theta \sin 2\zeta) + m_1 n_1 r_1 \cos \theta}. \quad (3)$$

Equation (4) gives the amplitude ratio f of the refracted SV wave when a P wave is incident in a liquid (oil) against a solid (aluminum).

$$f = \frac{2n_2/m_2 \sqrt{r_2 \sin 2\alpha \sin 2\beta} \cos \epsilon}{\cos \epsilon + n_2 r_2 \cos \alpha [1 - 2 \sin \beta \sin 2\beta (\cos \beta - 1/m_2 \cos \epsilon)]}. \quad (4)$$

Elastic Moduli

The elastic moduli for isotropic elastic materials are computed from the relations [5] given below:

$$\text{bulk modulus } K = \rho_2 (V_p^2 - 4/3\ V_s^2) \quad (5)$$

$$\text{bulk compressibility} = 1/K \quad (6)$$

$$\text{rigidity modulus} = \rho_2 V_s^2 \quad (7)$$

$$\text{Young's modulus} = 9\rho_2 V_s^2 m_3^2 / 3m_3^2 + 1 \quad (8)$$

$$\text{Poisson's ratio} = \tfrac{1}{2}[(m_1^2 - 2)/(m_1^2 - 1)]. \quad (9)$$

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